On a New Methodology for the Design and Operations Planning of Natural Gas Pipeline Networks

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Kewords: *Pipeline Design*

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ABSTRACT

This paper presents a new methodology to design natural gas pipelines over a long time horizon considering determining timely expansions by the addition of compressors. The problem is framed in the context of a capacity expansion problem and is solved using a non-linear mixed integer model. We compare our methodology to the current state-of-the-art methodologies, highlighting the limitations of the latter and showing the advantages of the former, especially in the case of branched networks.

1. INTRODUCTION

The economical design of pipelines is important because the cost for transmission and distribution is almost half of the total price the end user pays for natural gas. The United States consumes between 1.5 and 2.5 million MMcf of natural gas per month depending on the season, and this huge volume has to be transported somehow to the end user[1]. About 97 % of the natural gas consumed in the U.S. is produced at various places in North America. So, there is no hub where all the gas is produced, the producing sites are constantly changing their rates and producing status and new wells are also constantly being discovered. Currently, it is common to examine the feasibility of projects based on J-curves which analyze the Total Annual Cost per MCF transported versus the flow rate. This type of analysis can be sufficient for a simple project with no ramifications and a single pipe. But when analyzing a more complicated network, J-curves become exponentially more time consuming, and many times will not even give the correct optimum.

2. PROBLEM STATEMENT

We now discuss different cases:

a. Conventional structure

In this case the pipeline routes and compressors locations are predefined, but initial capacities and expansion capacities (size and timing) are not. Compressors are located only at the supplier points and or some consumer or recompression station points. We want to know the optimum pipeline diameters for each section and the compressors capacities at the suppliers. Branching and non branching structures can be analyzed. The following diagram serves as an example of the conventional design approach, from the planning stage to some time t*.
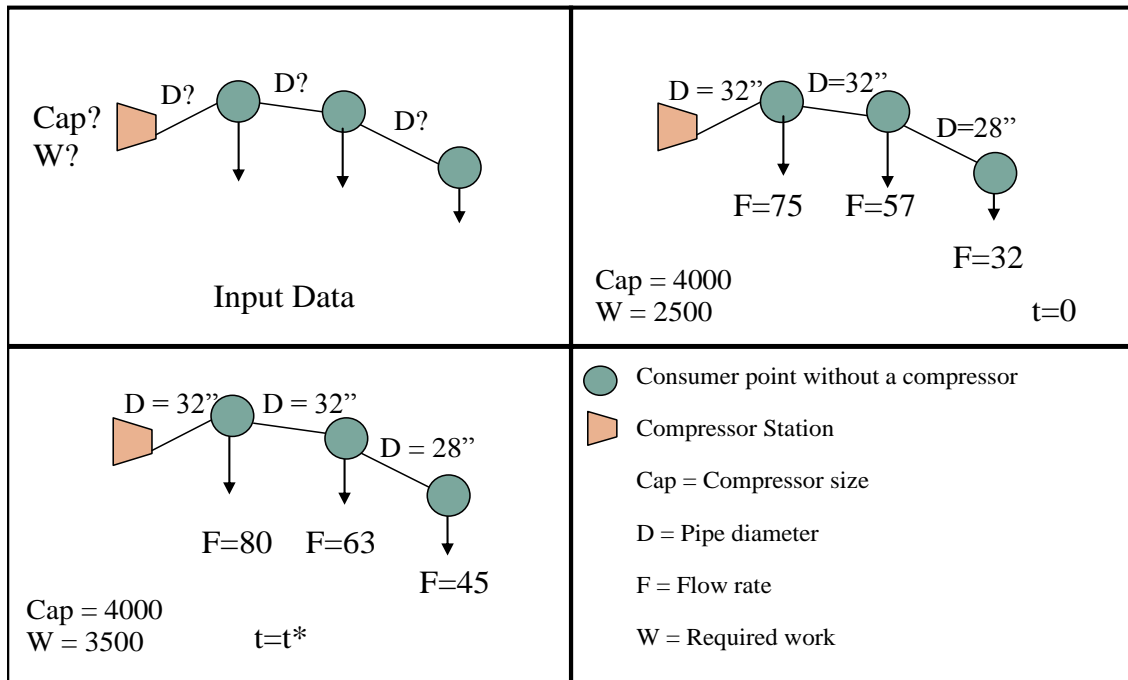
In the above example, the maximum compressor capacity and required diameters are computed. When the demand increases at time t*, the work of the compressor increases, while the capacity and diameters remain constant.

b. Compressor location

The pipeline routes are predefined. We want to know the pipeline sections diameters and where, when and with which capacity a compressor should be installed.
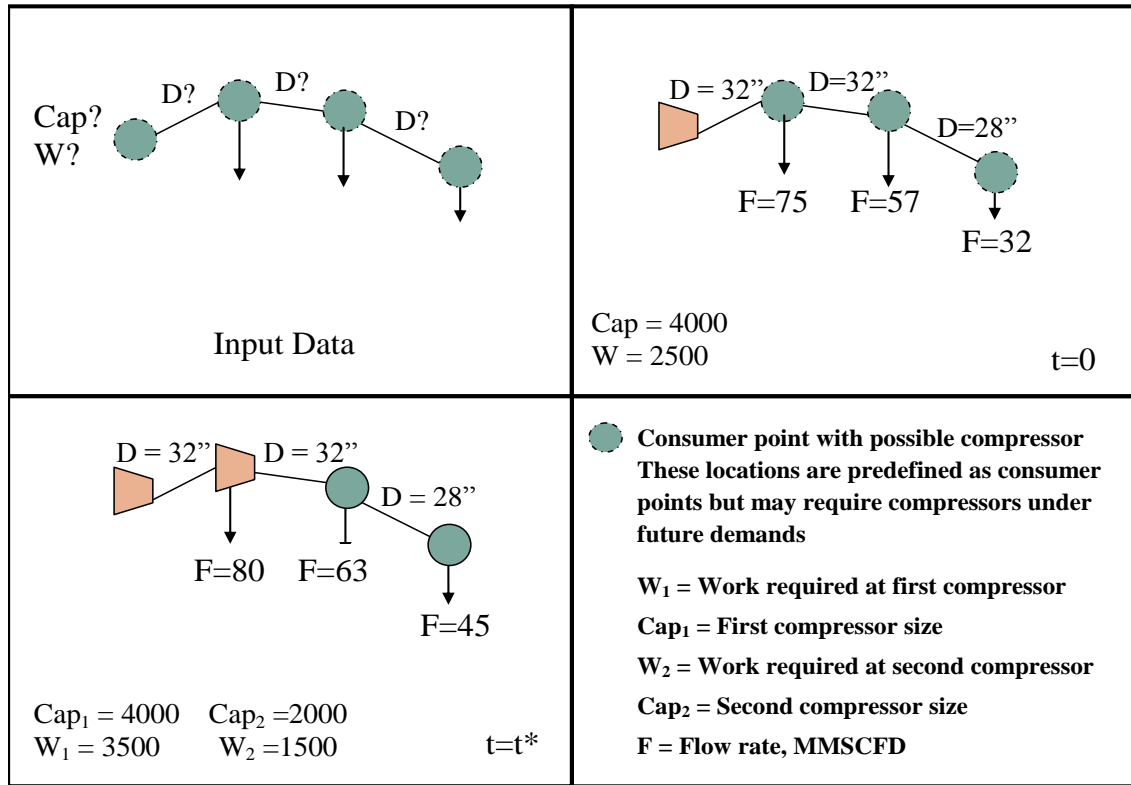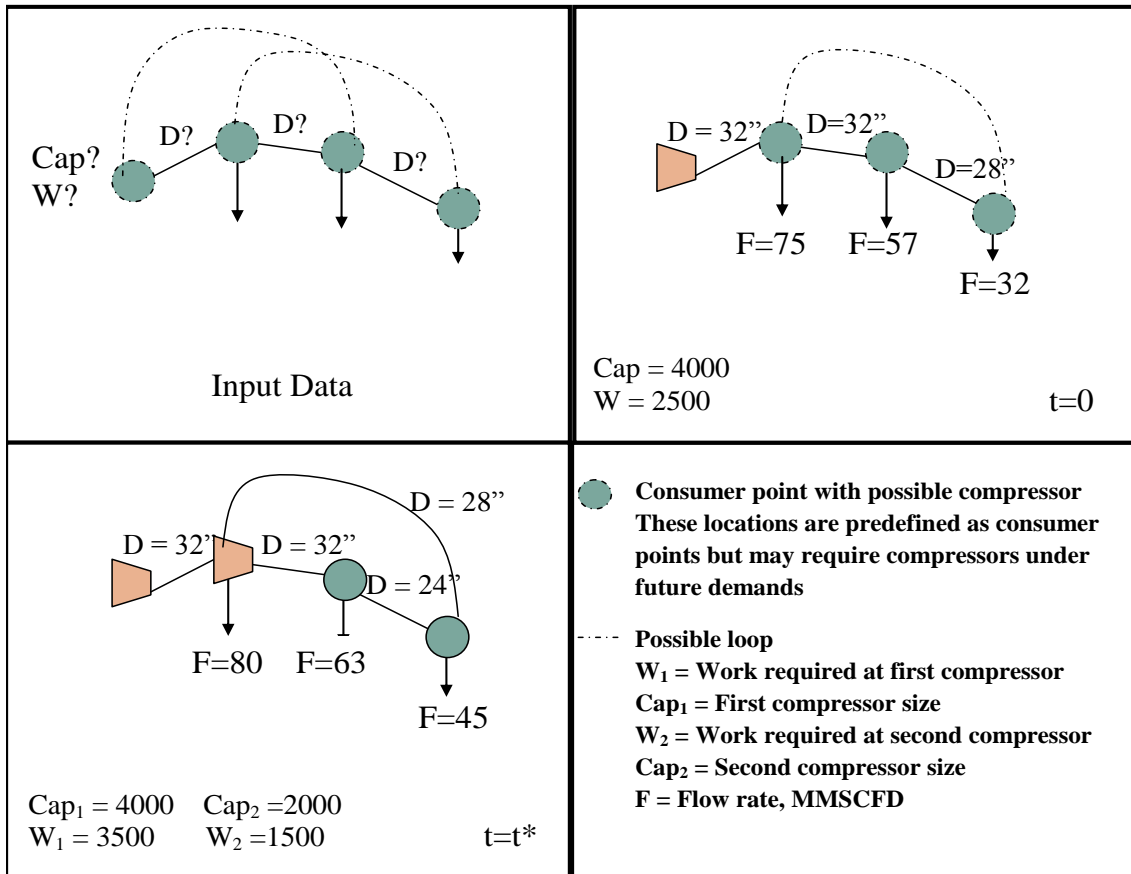

During the design phase, the consumer points are predefined. It is unknown if a compressor is required at any of these points. By time t=0, a compressor is required at the supply point, yet the possible increase in demand may require further addition of compressors at the consumer points. At time t=t*, a second compressor is added, and all other consumer points are no longer available for adding compressors.
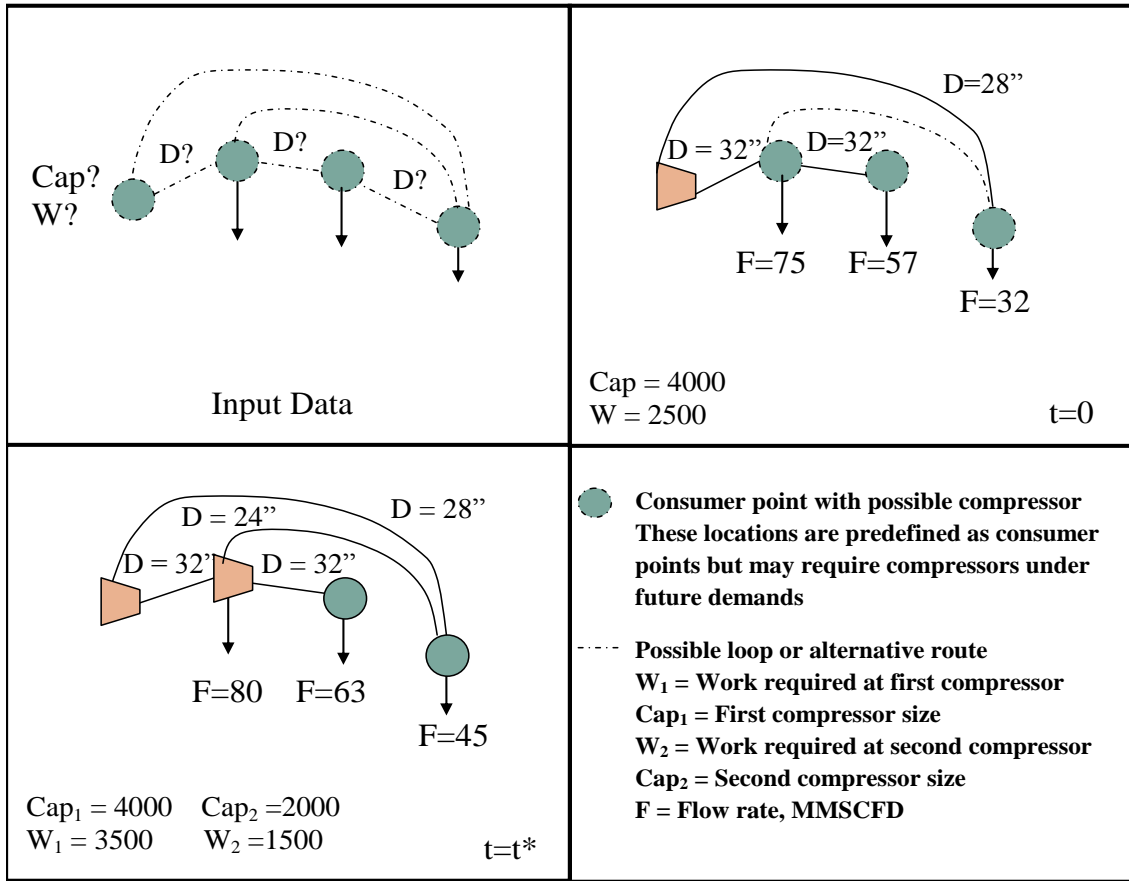
c. Loop and compressors locations

The pipeline routes are predefined. We want to know the pipeline diameters; where, when and with which capacity a compressor should be installed; and, where, when and which capacity loops should be added. During the design phase, two possible loops are considered in this example. The route has already been determined, but an increase in

demand may require one or both of the loops at a later time. At time t=0, adding the supply compressor determines that only one loop will be further considered.   The remaining consumer points are still available for adding compressor as required. At time t=t* the increase in demand requires greater work at the compressor. The pipe diameters at t=t* are slightly varied from those of t=0. The maximum required diameter must be chosen for operation.

Input Data

$D = 32"$ $D=32"$

$D=28?"$

$F=75$ $F=57$

$F=32$

$Cap = 4000$
$W = 2500$ $t=0$

$D = 28"$

$D = 32"$ $D = 32"$

$D = 24"$

$F=80$ $F=63$

$F=45$

$Cap_1 = 4000$ $Cap_2 =2000$
$W_1 = 3500$ $W_2 =1500$ $t=t*$

Consumer point with possible compressor
These locations are predefined as consumer points but may require compressors under future demands

Possible loop
W_1 = **Work required at first compressor**
Cap_1 = **First compressor size**
W_2 = **Work required at second compressor**
Cap_2 = **Second compressor size**
F = **Flow rate, MMSCFD**

d. Alternative routes, loops and compressors locations

Only the consumers and suppliers distances are predefined. We want to know where, when and with what capacity pipelines (including loops) and compressors should be installed. The possible loops and alternative routes are shown in the input data. At time t=0, an alternative route is chosen, but the consumer points are shown as having the option of adding a compressor. By t=t*, a second compressor is added and the other consumer points are not available for compressor addition.

Cap? W? D? D? D? D? **Input Data**	D=28" D = 32" D=32" F=75 F=57 F=32 Cap = 4000 W = 2500 t=0
D = 24" D = 28" D = 32" D = 32" F=80 F=63 F=45 Cap₁ = 4000 Cap₂ =2000 W₁ = 3500 W₂ =1500 t=t*	**Consumer point with possible compressor** These locations are predefined as consumer points but may require compressors under future demands ----- **Possible loop or alternative route** W₁ = Work required at first compressor Cap₁ = First compressor size W₂ = Work required at second compressor Cap₂ = Second compressor size F = Flow rate, MMSCFD

In this paper only cases a and b will be approached. The others are subject for further study.

3. PIPELINE DESIGN BASICS

Typically, to calculate the pressure drops, the literature recommends using the Panhandle A equation (for Reynolds numbers between 5 and 11 million)

$$Q = 435.87 \times E \times \left(\frac{T_b}{P_b}\right)^{1.0788} \left(\frac{P_1^2 - e^s P_2^2}{G^{0.8539} T_f L_e Z}\right)^{0.5394} D^{2.6182}$$

(1)

and the Panhandle B equation (suitable for Reynolds numbers between 4 and 40 million)[4].

$$Q = 737 \times E \times \left(\frac{T_b}{P_b}\right)^{1.02} \left(\frac{P_1^2 - e^s P_2^2}{G^{0.961} T_f L_e Z}\right)^{0.51} D^{2.53}$$

In these equations,

Q volume flow rate (SCFD)

E the Pipeline efficiency

P_b the base pressure (psia)

T_b the base temperature (°R)

P_1 the upstream pressure (psia)

P_2 the downstream pressure (psia)

G the gas gravity

T_f the average gas flow temperature (°R)

L_e the equivalent length of pipe segment (miles)

Z the gas compressibility factor (dimensionless)

D the pipe inside diameter (in)

e = base of natural logarithms (e=2.718…)

s = elevation adjustment parameter, dimensionless

The elevation adjustment parameter is found from the following equation:

$$s = 0.0684G \left(\frac{H_2 - H_1}{T_f Z} \right)$$

Where

H_1 is the upstream elevation (m)

H_2 is the downstream elevation (m)

The panhandle equations were derived from simplifications of the Bernoulli equation. In its most inclusive form, the Bernoulli is as follows:

$$Q_b = \pi \sqrt{\left(\frac{g_c R}{1,856}\right)} \times \frac{Z_b T_b}{P_b} \sqrt{\left(\frac{P_1^2 - P_2^2 - \frac{58G \times \Delta H \times P_{ave}^2}{R T_{ave} Z_{ave}}}{Z_{ave} T_{ave} GL} \right)} \times \sqrt{\left(\frac{1}{f}\right)} \times D^{2.5}$$

In order to arrive at the panhandle equations above, specific transmission factors are used. For the Panhandle A equation, the transmission factor is $6.872(\text{Re})^{0.0730}$. For the Panhandle B equation, the transmission factor is $16.49(\text{Re})^{0.01961}$. The constant values

of the panhandle equations are dependent on an assumed viscosity, as well as the assumption that $P_1^2 - P_2^2 \gg \dfrac{58G \times \Delta H \times P_{ave}^2}{RT_{ave}Z_{ave}}$.

Q_b is assumed to be at base conditions, with a base temperature of 520ºR and a base pressure of 14.7 psia.

Besides these pressure drop correlations, simulation software can also be used. Here we compare the results using the Panhandle correlations to the ones found using simulations from Pro/II. The simulations used the Beggs-Brill-Moody pressure drop correlation, which is good for mostly vapor hydrocarbon systems with small elevation changes, very similar to this situation. We used the Soave-Redlich-Kwong thermodynamic equation of state, which is suitable for hydrocarbon systems such as the one being examined. A negligible pipe roughness was assumed, along with a length of 120 miles, a composition listed in Table 1, an outlet pressure of 800 psig, an inlet temperature of 60° F, a viscosity of 7.53 x 10^{-7} lb/ft/s, and a pipeline efficiency of 0.92[2,3,4]. An overall coefficient of heat transfer was assumed to be 0.3 Btu/hr-ft^2-°F based on rough calculations using the correlation by Churchill and Chu[6].

Table 1

Natural Gas Composition Used[5]	
Natural Gas Component	Mole Fraction
C_1	0.949
C_2	0.025
C_3	.002
N_2	0.016
CO_2	0.007
C_4	0.0003
iC_4	0.0003
C_5	0.0001
iC_5	0.0001
O_2	0.0002

The two independent methods, Pro-II simulation and analytical correlations, were used to predict these pressure drops. Both methods were analyzed by the same parameters: flow rate, pipe length, pipe diameter, and downstream pressure. The two methods supplied an estimate of the outlet compressor pressure, which produced two distinct pressure drops for each comparison. The comparison was then made for different pipe diameters and flow rates.

Figure 1: Comparison of pressure drops found using Panhandle equations and Pro/II – a) No elevation. b) Considering elevation.
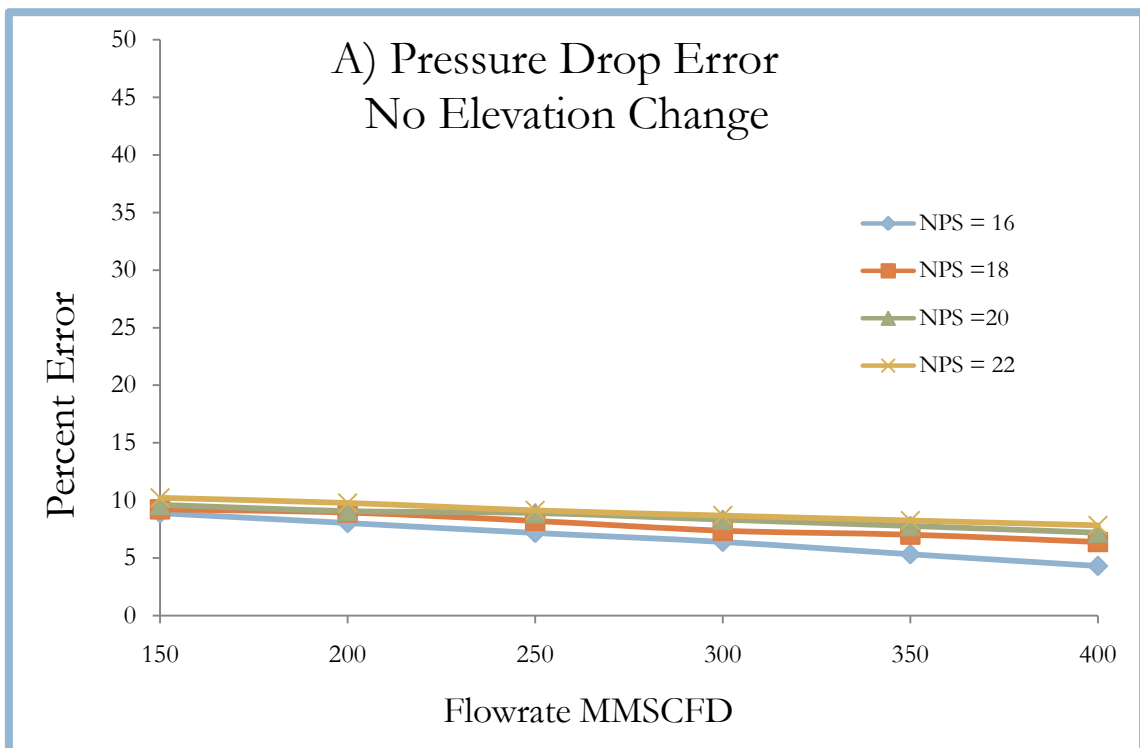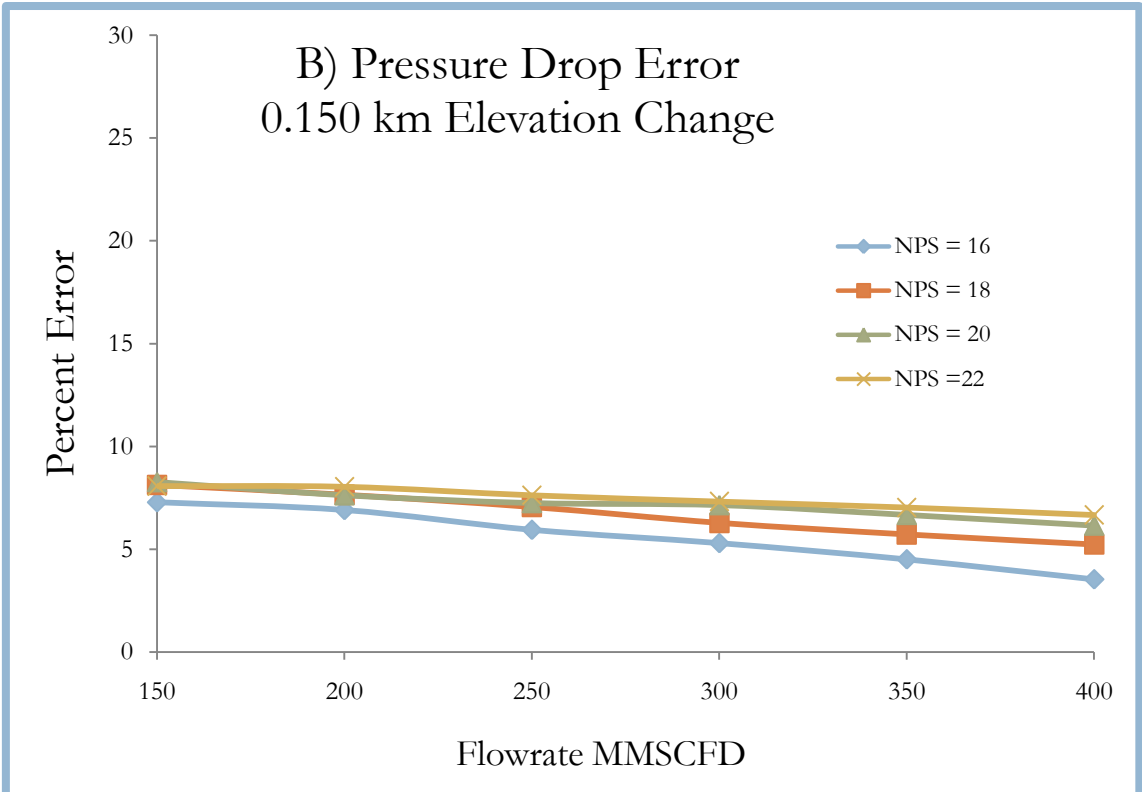

A) Pressure Drop Without Elevation Change
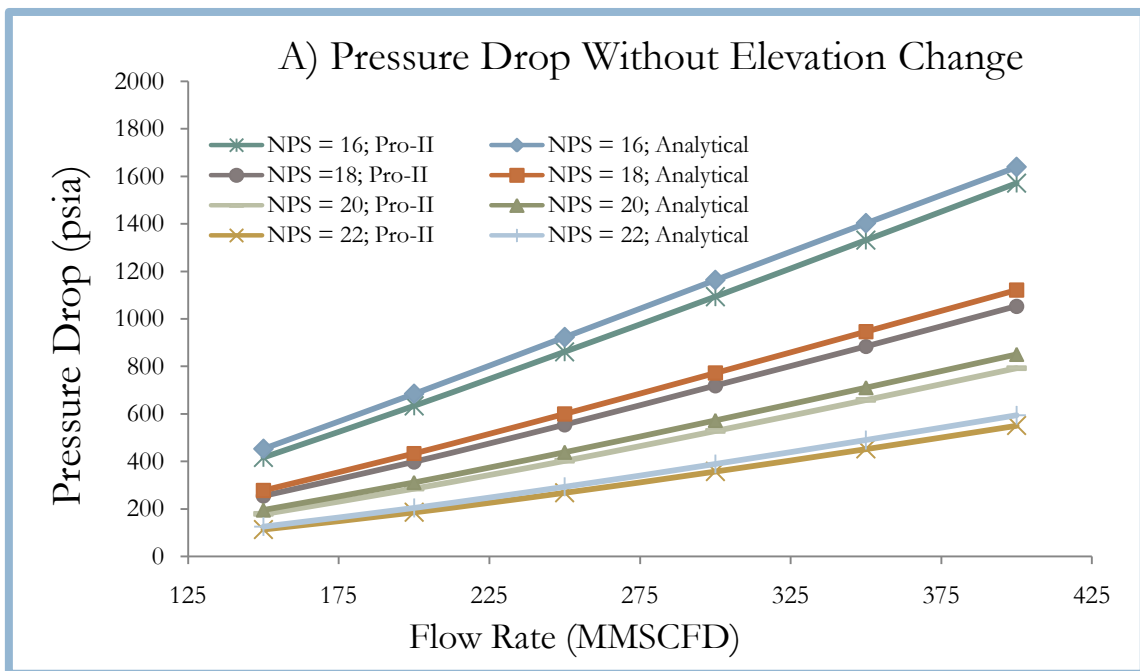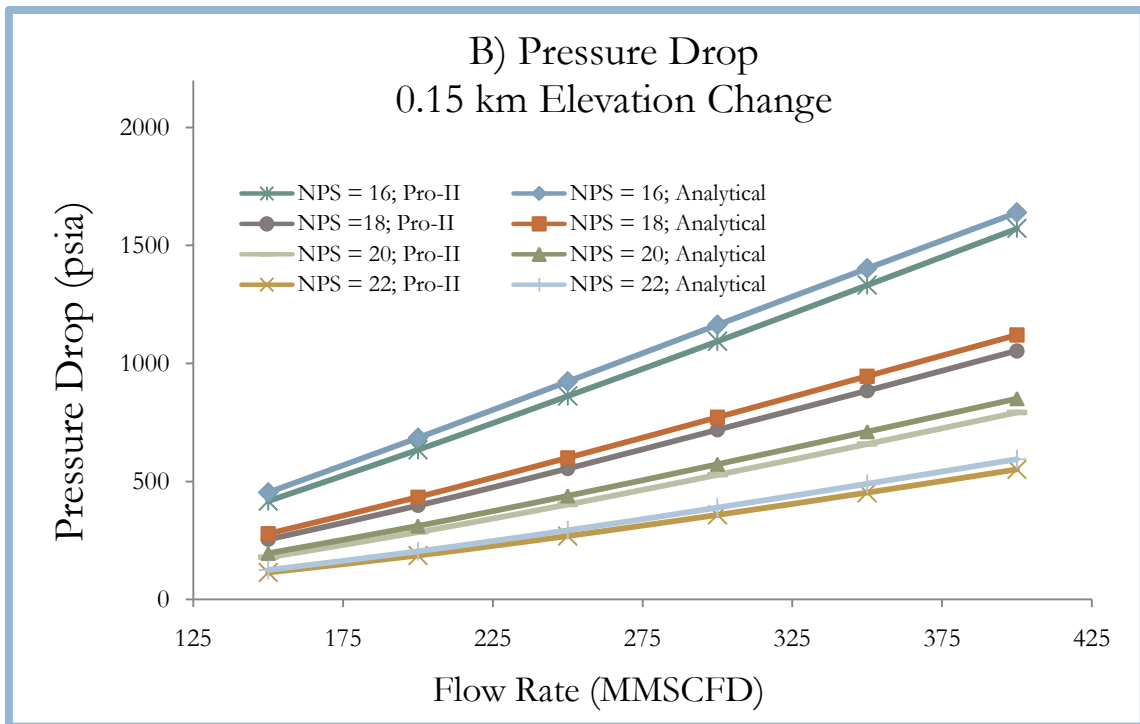
Figure 2: Comparison of pressure errors with respect to Pro-II using a pipeline efficiency of 0.92 – a) No elevation. b) Considering elevation.

B) Pressure Drop Error
0.150 km Elevation Change

In order to analyze the effects of pipeline efficiency on the accuracy of the Panhandle equations, the pipeline efficiency was changed using the solver function in Microsoft Excel to minimize the sum of the errors graphed below in Figure 3.

Figure 3: Comparison of pressure errors with respect to Pro-II using an adjusted pipeline efficiency – a) No elevation. b) Considering elevation.
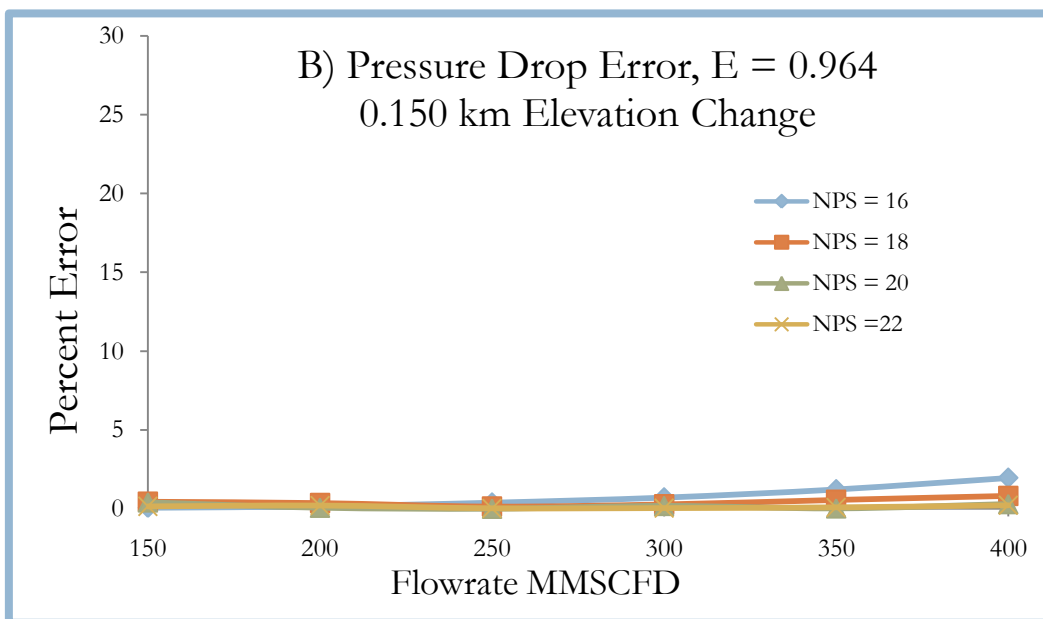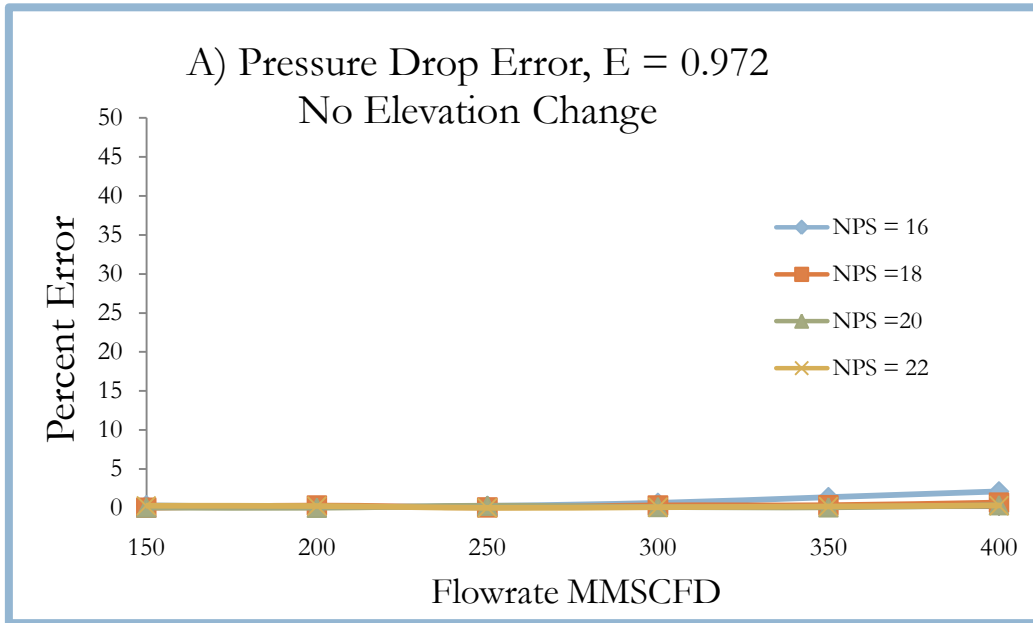
By using solver it was possible to minimize the pressure drop error to around 2% or less; however, this still requires the use of a simulator to find a more accurate Panhandle equation. It seems redundant to use a simulator to adjust a correlation that should be able to provide the pressure drop without the use of any simulator. Therefore, for our J-curves we use Pro-II to determine the pressure drop across pipelines.

If the Panhandle equation was used, then the following equation can be used to calculate the required horsepower of the compressor

$$HP = 0.857 \left(\frac{\gamma}{1-\gamma}\right) QT_1 \left(\frac{Z_1 + Z_2}{2}\right) \frac{1}{n_a} \left[\frac{P_2}{P_1}\right]^{\frac{\gamma-1}{\gamma}}$$

(3)

Where

γ is the ratio of specific heats of gas, dimensionless

T_1 is the suction temperature of gas (°R)

P_1 is the suction pressure of gas (psia)

P_2 is the discharge pressure of gas (psia)

Z_1 is the compressibility of gas at suction conditions

Z_2 is compressibility of gas at discharge conditions

n_a is the compressor adiabatic efficiency[4]

Typically, the compressors used are centrifugal which operate at a maximum of 80% efficiency.

4. CONVENTIONAL DESIGN APPROACH

In this section we review the existing recommended procedure for optimizing a pipeline network. Traditionally, pressure drop correlations are used to determine the work required by the compressor. The usual situation described in the literature is that of a single pipe with no branches and one compression station at the beginning. Costs are then directly estimated for each diameter and translated into what is known as a J-Curve, which is a graph of the Total Annual Cost ($ per Mcf delivered) versus flow rate (Figure 1). The range of flow rates can be a range of expected flows due to increased future demands, stationary changes, or even day vs. night fluctuations. To obtain the total cost for each flow rate, capital and operating costs are used. The capital costs are computed by annualizing the compressor as well as the pipe fixed capital investment.

Operating costs are related to the power consumed, as well as maintenance and overhead costs, such as any materials required during daily operation or labor expenses not incurred during installation. Interstage cooling costs are included as an operating expense, if required, as well as routine maintenance to compressor stations or pipeline.
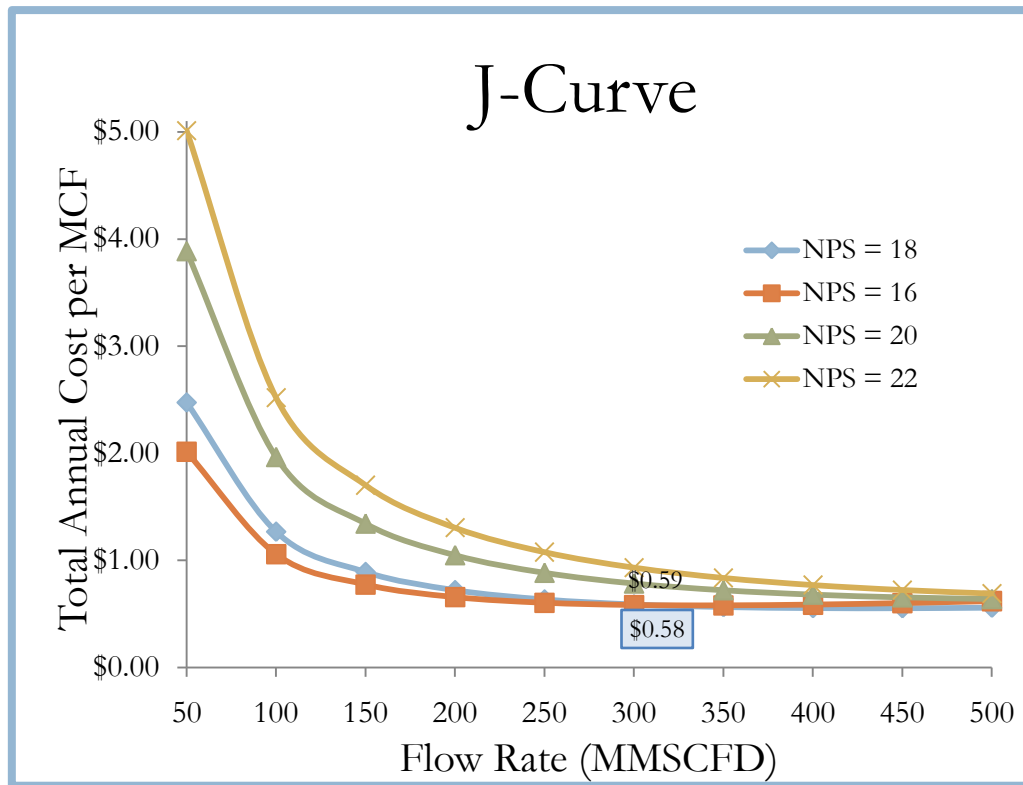

Figure 4: A 'J-Curve' showing the total pipeline cost per Mcf versus flow rate for four different pipe sizes

The pipe material cost is given by[4]:

$$PMC = \frac{10.68(D - T)TLC \times 5280}{2000}$$

where:

      D is the diameter (in)

      T is the wall thickness (in)

      L is the pipe length (mi)

C is the material cost ($/ton)

In turn, the pipe installation cost is found using the values given in Table 2[4]:

Typical Pipe Installation Costs	
NPS	Cost, $/in-dia/mi.
16	14,900
18	17,500
20	20,100
22	27,025

Table 2: Pipeline Installation Costs

The total pipe cost = PMC + Installation + Wrapping & Coating, where wrapping and coating are usually 5% of the PMC. The cost of the compressor station labor and materials is found from[7]

$$Cost = 0.79(1799.7 \times HP + 3 \times 10^6)$$

The amount of fuel gas required to operate the compressor station depends on the amount of horsepower the compressor is operating at. The following equation depicts the relationship between fuel gas requirements and the operating horsepower at the compressor station at three specific pipe diameters[4].

$$Y = 0.0002x + 0.0006$$

The data points used to derive the linear relationship were obtained from Gas Pipeline Hydraulics, by Menon. The equation allows the fuel requirement to be found for a wide range of compressor sizes.

The total capital costs are then calculated from equipment, miscellaneous costs, right of way, acquisitions, SCADA, and telecommunication systems, and other initial costs. The total annual capitalized costs include capital and annual fuel costs.

The following derivation is for the Total Annual Costs, which is the annual cost the project will cost at an interest rate of 8%. To find the annualized capital cost of the

fuel and the equipment, start with the present worth (P) for a stream of equal annual cash flows (A) over the course of n years at an interest rate of i.

$$P = A\left[\frac{1}{1+i} + \frac{1}{(1+i)^2} + \cdots \frac{1}{(1+i)^{n-1}} + \frac{1}{(1+i)^n}\right]$$

Equation 1

Multiply both sides by $\dfrac{1}{1+i}$

$$\frac{P}{1+i} = A\left[\frac{1}{(1+i)^2} + \frac{1}{(1+i)^3} + \cdots \frac{1}{(1+i)^n} + \frac{1}{(1+i)^{n+1}}\right]$$

Equation 2

Subtract Equation 1 from Equation 2.

$$\frac{-i}{1+i}P = A\left[\frac{1}{(1+i)^{n+1}} - \frac{1}{1+i}\right]$$

Then solve for P and simplify.

$$P = \frac{A}{-i}\left[\frac{1}{(1+i)^n} - 1\right] = A\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$$

Then solving for A yields the amount needed annually to recover the present worth of the capital cost (P) of a project that is expected to last n years at an interest rate of i.

$$A = P\left[\frac{i(1+i)^n}{(1+i)^n - 1}\right]$$

Next, the following equation is used to find the total annual cost per MCF[4]

$$TAC\ per\ MCF = \frac{TAC \times 1000}{Q \times 350 \times 10^6}$$

Where Q = flow rate, MMSCFD assuming 350 operating days per year.

In this procedure, the optimum pipe diameter is identified by corresponding to the curve with the lowest cost, which ideally should also have a flat profile over the range of flow rates of interest. Figure 6 shows curves for a single, unbranched pipeline with a feed at the inlet and a single consumer point at the outlet.

To exemplify the use of J-Curves we consider the design of a pipeline between a supplier point and consumer point (single pipe case), as seen in Figure 5. In the simulations, the compressor power was varied so that the pressure at the pipe outlet was 800 psig for the different pipe diameters and flow rates.
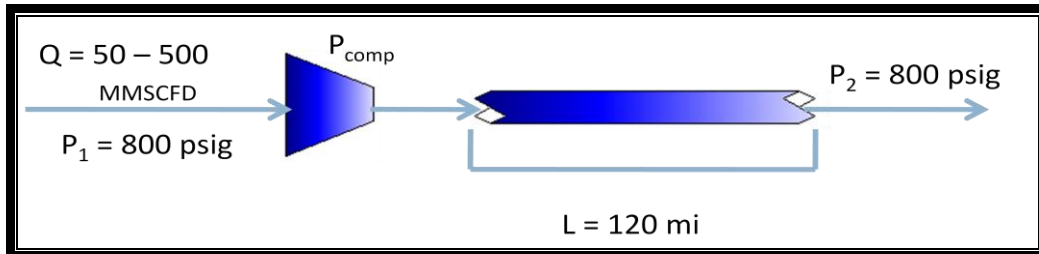

Figure 5: Simulation setup used to create J-curves in Figure 6.
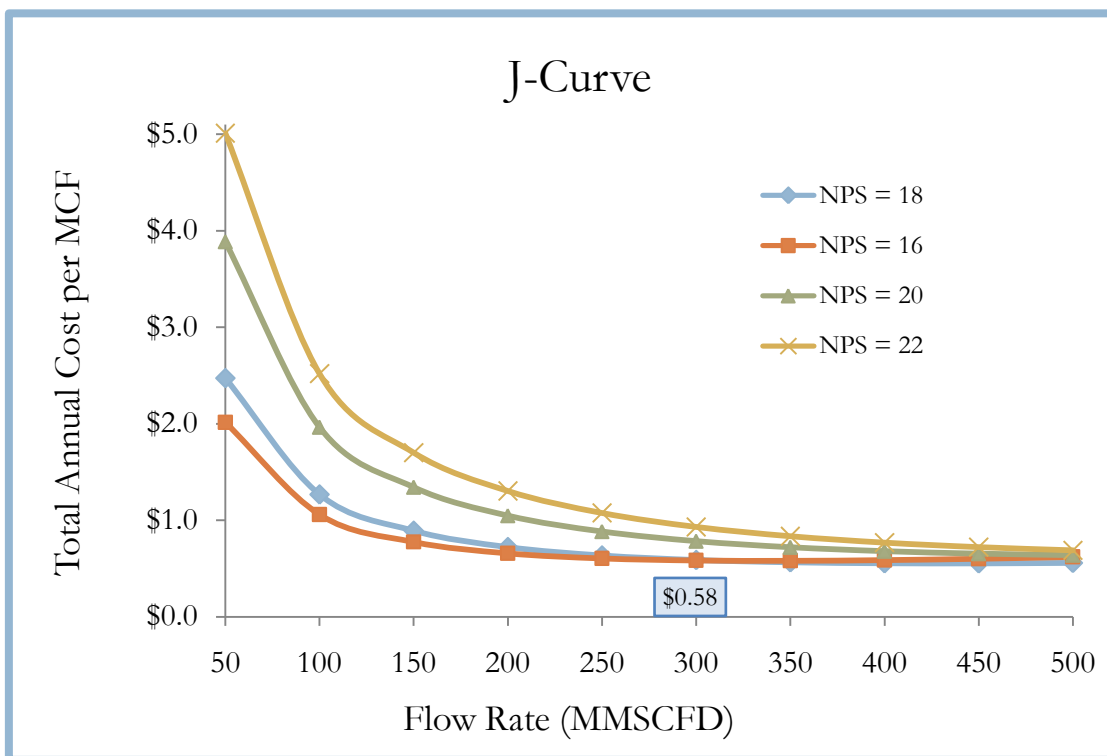

Figure 6: J-Curves for the design of a single pipe segment.

For this situation, if the system was intended to have a flow rate of 300 MMSCFD, then a pipe diameter of 16 inches would be the economic optimum with a Total Annual Cost of $ 0.58 per MCF delivered.

Limitations of the Existing Procedure

Several limitations to using J-Curves to select a pipe diameter and compressor size can be identified.

1. The J-Curve is designed with a different compressor at each point along the graph, so when a compressor is selected, the only accurate cost is at a single point. A more accurate method is to select a compressor, and find the new costs for several flow rates using a single compressor size. By selecting a compressor, the capital cost of the compression station is fixed for all flow rates. However, different flow rates require different powers at which the compressor operates resulting in varied operating and fuel costs. In addition, the compressor operates at different efficiencies when it is operated at different flow rates than that for which it was designed.

2. The Panhandle equations have some errors. Simulation software can be a more accurate and direct estimation of these pressure drops and compressor power requirements. The Panhandle equations do not account for pipe roughness in their pressure drop analysis[4].

3. The range of flow rates used and the criterion of choosing the lowest curve works well when the curves do not cross. Otherwise, if the curves cross, one needs to pay more attention to the frequency at which the pipeline will operate for each flow rate. Although this is not described in the open literature, we assume that some consideration is given to this issue in practice.

4. The J-curve procedure seems straightforward for one pipe and one compressor. It is unclear how to use it for a long pipeline with multiple compressors, branched pipelines, gathering and distribution systems, etc. It seems that each section is treated separately after the location of the compressors is decided somehow, without looking at how one set of decisions for one section affects the other. We assume that current practice does this but we are unaware of existing procedures to decide the compressors locations.

Our Extensions of the Existing Procedure

To make fair comparisons with the J-Curve procedure, a few modifications have been introduced.

1. The use of the Panhandle equation is substituted by computer simulations using more accurate pressure drop correlations for the subsequent J-curves.

2. For each J-curve, one compressor is picked, such that the same compressor can handle all flow rates. This is usually the compressor at the largest flow rate

3. Efficiency changes in the selected compressor due to operating at different flow rates are taken into account.

Illustration

The modified J-Curves are illustrated for a one pipe network and compared with the above presented literature conventional J-curve with changing compressor sizes. First a constant compressor efficiency of 0.8 is assumed. The extended curves represent a more realistic prediction of the cost of operating a pipeline by fixing the compression station installed power. The curves are only as long as they are, because above the end of the line is the stonewall point, and on the lower end is the surge point. Outside of these bounds the compressor acts unpredictably and very inefficiently.
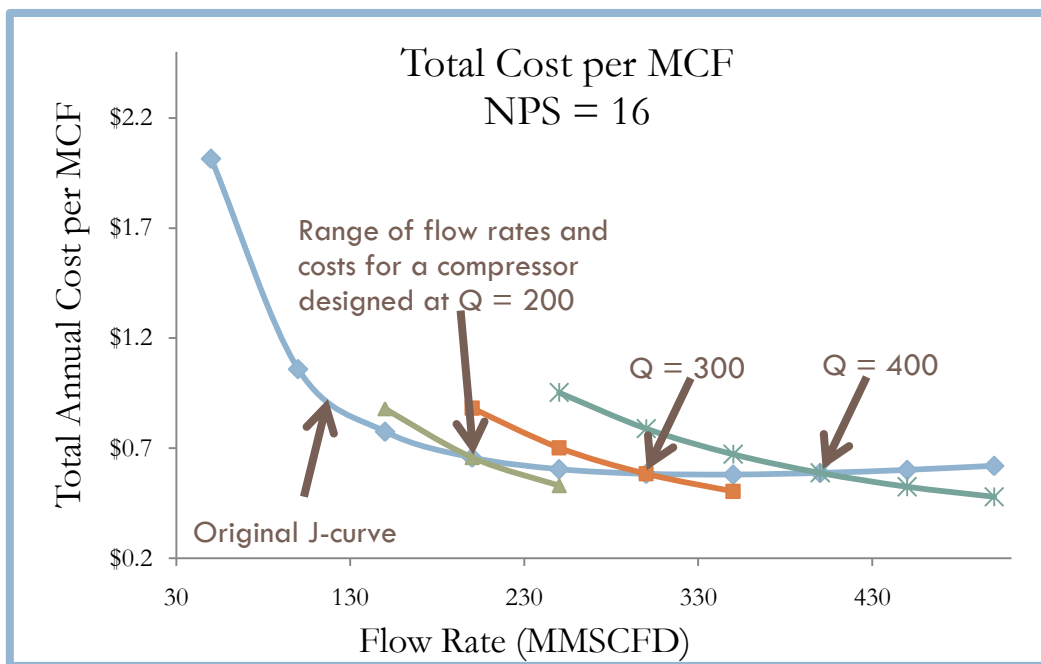
Figure 7: J-Curves for the design of a single pipe segment considering fixed compressor sizes.

While both graphs indicate the same optimum, the evidence is less convincing in this extended curve. The total annual costs change significantly with flow rate, which is markedly different from the conventional curve. However, neither this extended graph nor the conventional J-curve considers the effects of flow rate on the compressor efficiency. When compressors operate above or below their design capacity, their efficiency decreases, which requires larger amounts of power for operation.

Figure 8 uses efficiency estimations based to take into account decreased efficiencies of flow rates higher and lower than the compressor was designed to process[8]. Figure 7 shows only the flow rates that the selected compressor can achieve, providing a more realistic look at what is possible with the selected compressor.
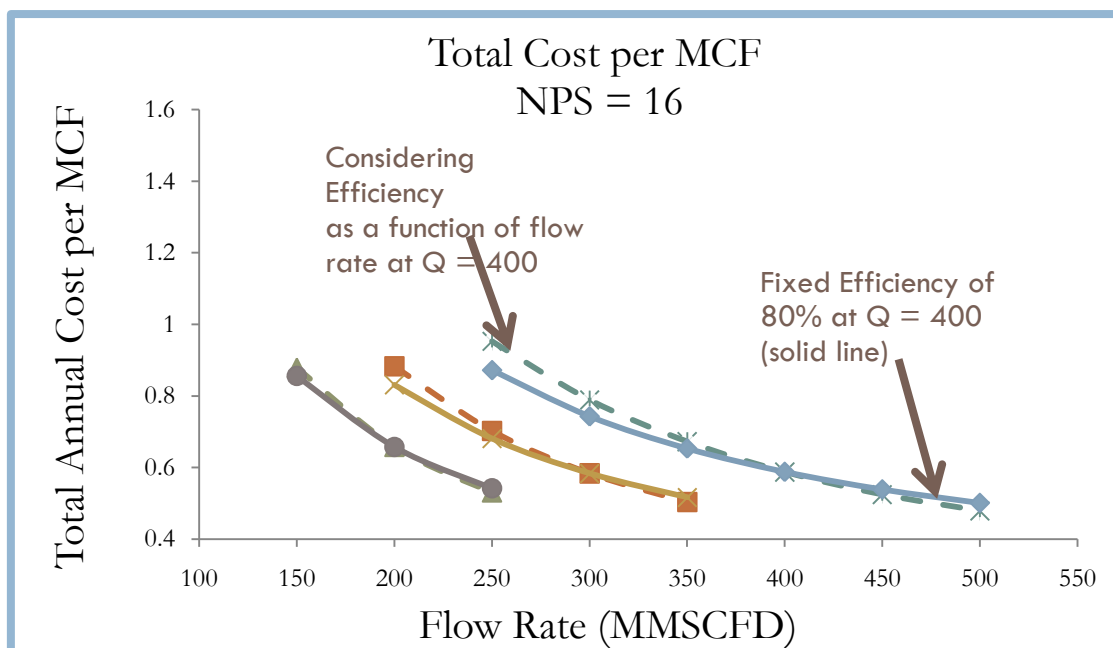

Figure 8: J-Curves for the design of a single pipe segment for different compressors sizes and considering efficiency changes.

In order to analyze the effects of the efficiency analysis, the graph below compares the 'Extended J-Curves' for a pipe size of NPS 18 with and without

efficiency. The TAC for each curve is consistently higher when considering efficiency. This is expected, since the actual work required is greater than the ideal work.

We now extend the method to a two compressor case with one consumer point half way. J-curves can be constructed, but now the intermediate pressure P_3 is a variable. Pro-II simulations were run at several diameters for varying flow rates, and P_3 pressures.
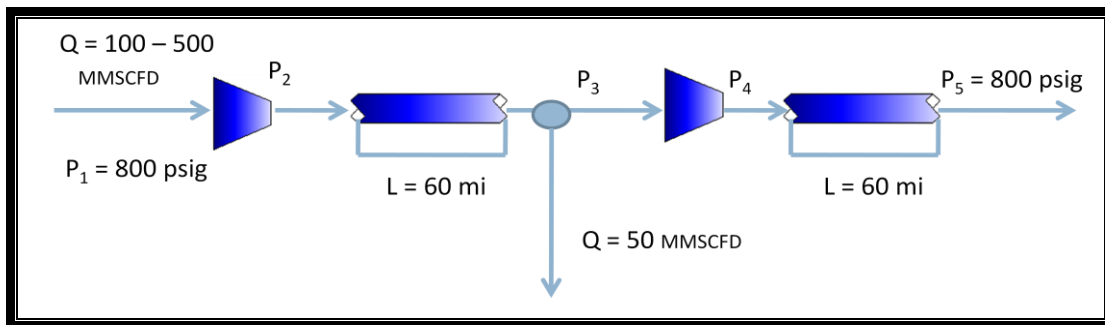
Q = 100 – 500 MMSCFD

P_2

P_3

P_4

P_5 = 800 psig

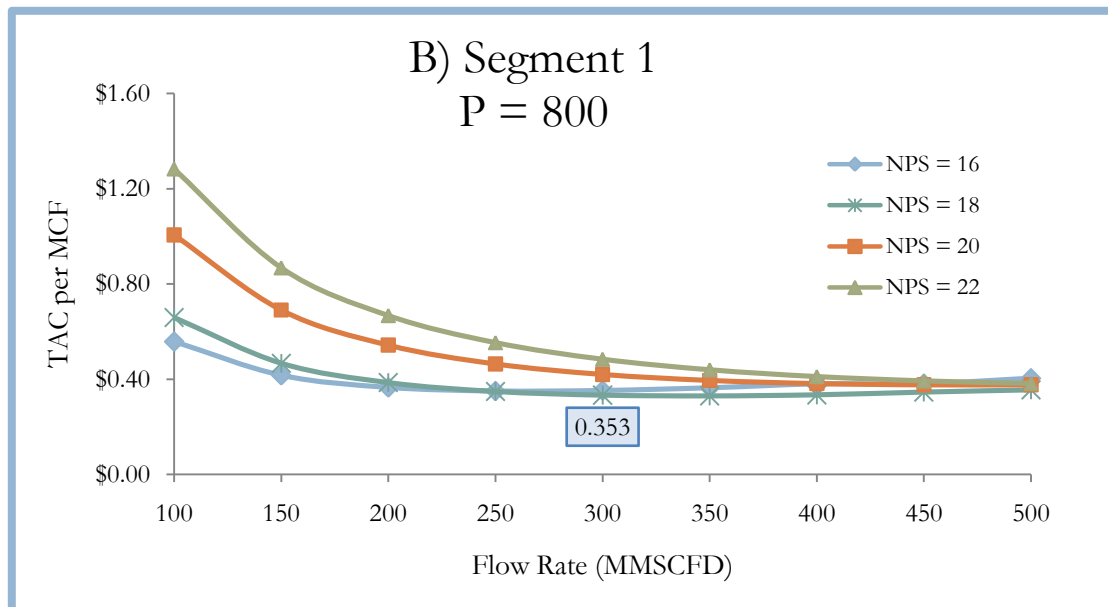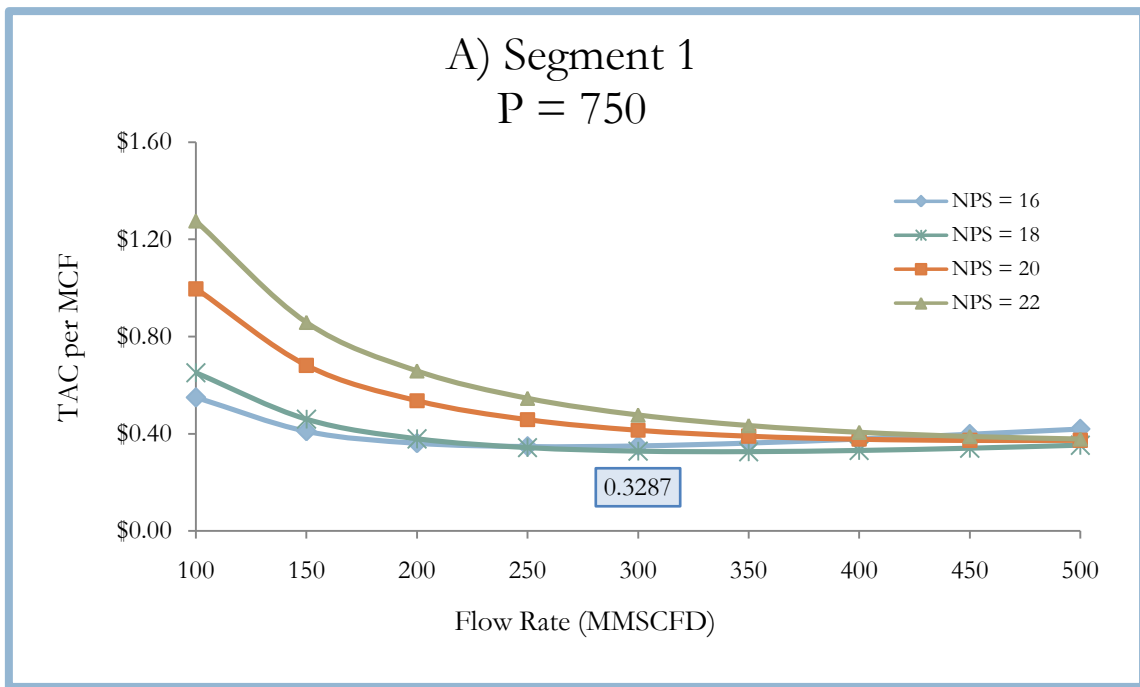P_1 = 800 psig

L = 60 mi

L = 60 mi

Q = 50 MMSCFD

Figure 9: Two segment pipeline with a consumer point in the middle.

In a complex network, Pro-II simulations paired with J-curve analysis allows each segment to be optimized. However, when designing the network, the optimum arrived at by picking the pressure parameter P_3 based on one segment can lead to an incorrect optimum (this case is outlined below the graphs). In this two-segment pipeline, three pressures were chosen for analysis: P_3 = 750, 800, and 850 psig. When these values were used (using Pro-II) for four different pipe diameters, their resulting pressure drops and compressor power requirements were used to construct the J-curves shown in Figure 10.

Figure 10: J-Curves for the design of segment 1. a) P_3=750psi; b) P_3=800psi; c) P_3=850psi.

Optimizing Segment 1 at each pressure
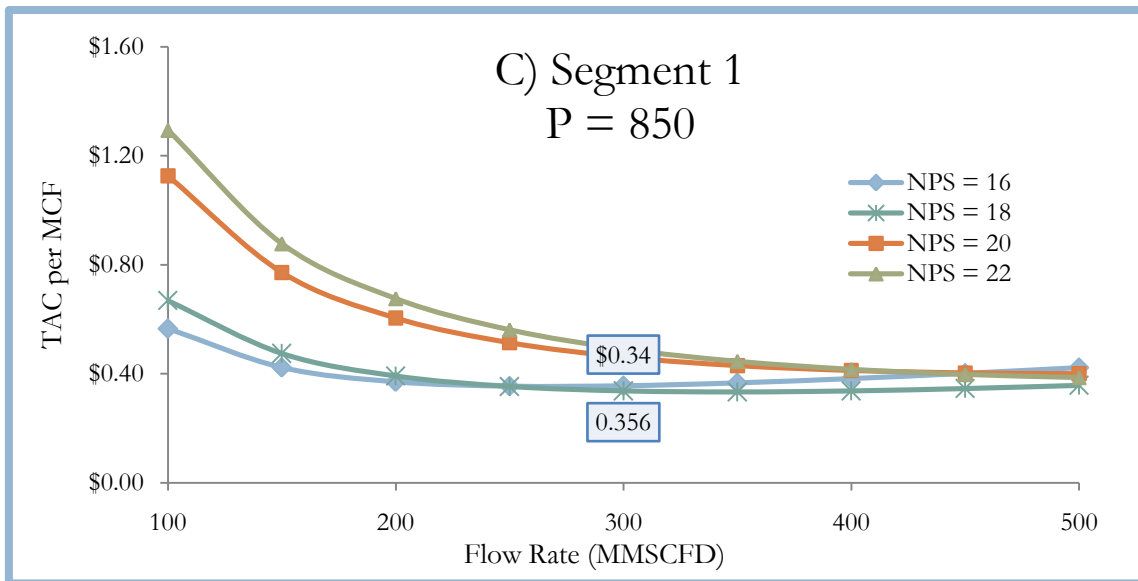
A) Segment 1
P = 750

B) Segment 1
P = 800

C) Segment 1
P = 850

Considering each of the three pressures and each diameter for segment 1, the optimum pressure was found to be 750 psig, and the best pipe diameter was 18 inches.

Figure 11: J-Curves for the design of segment 2. a) P_3=750psi; b) P_3=800psi; c) P_3=850psi

A) Segment 2
P = 750

B) Segment 2
P = 800

NPS = 16
NPS = 18
NPS = 20
NPS = 22

0.2903

0.293

C) Segment 2
P = 850

NPS = 16
NPS = 18
NPS = 20
NPS = 22

0.2791

0.2801
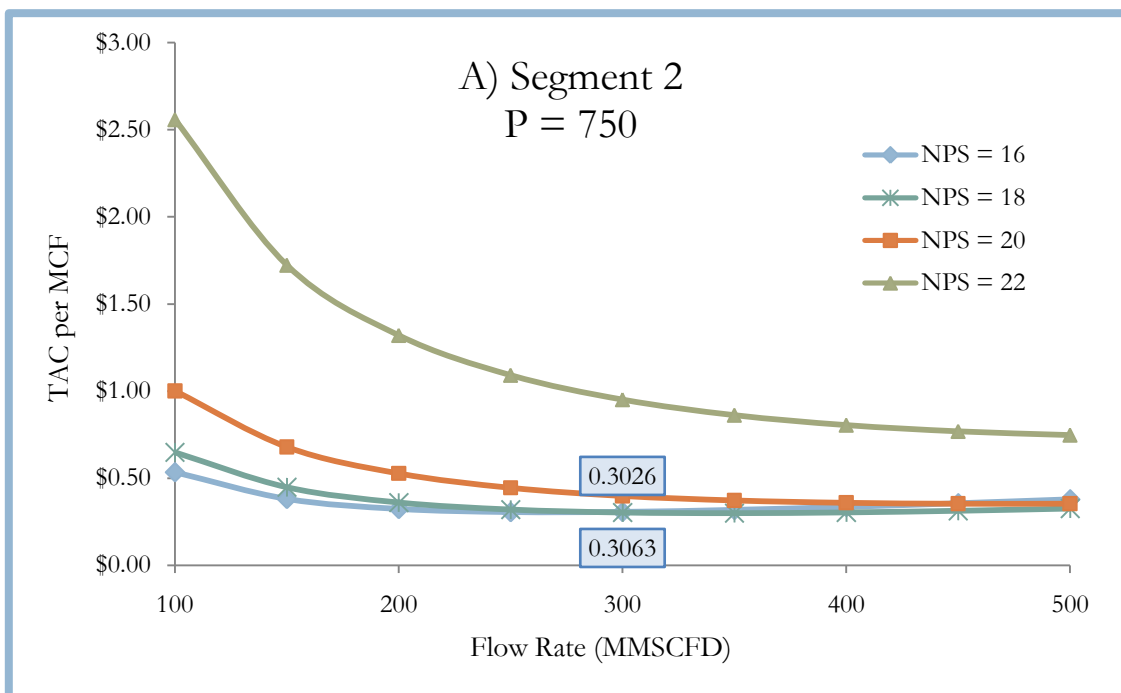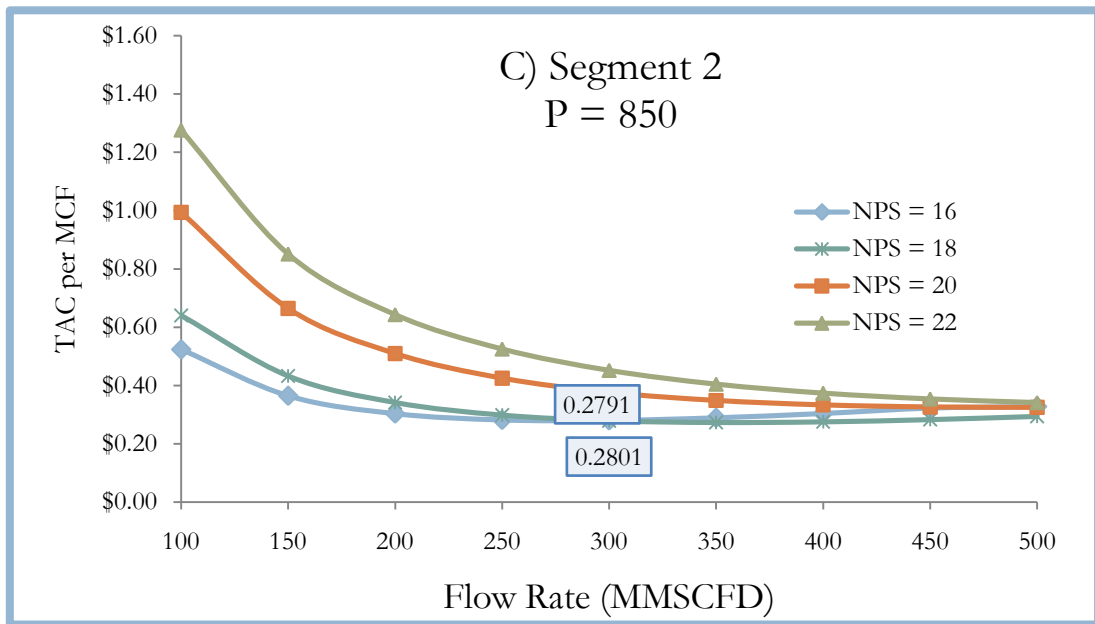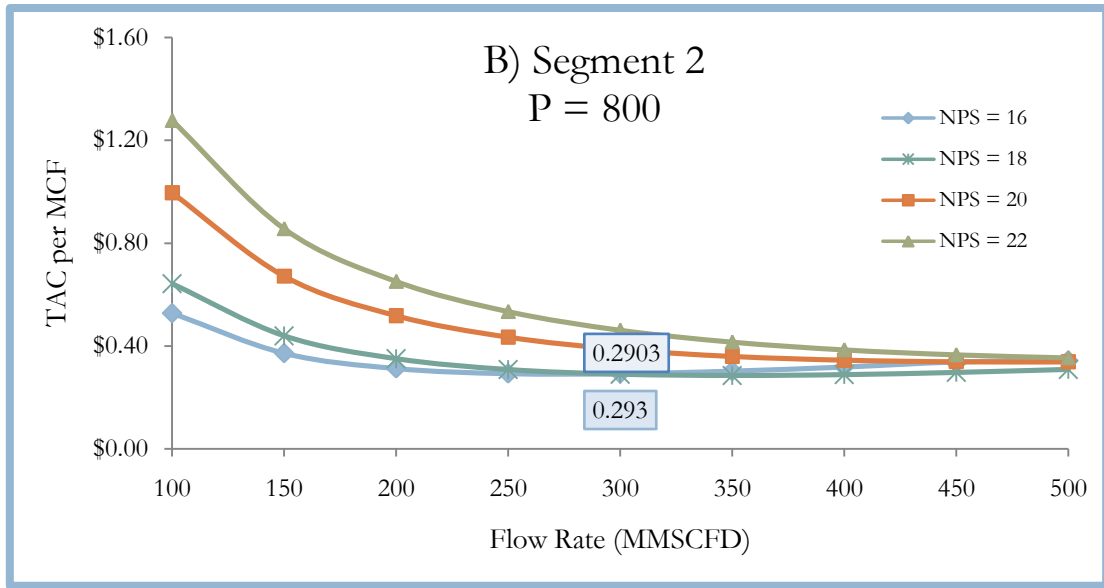
Considering each of the three pressures and each diameter for segment 1, the optimum pressure was found to be 850 psig, and the best pipe diameter was 18 inches.

Figure 12: Combined J-Curves (segment 1 + segment 2). a) Overall optimum found by optimizing segment 1 first; b) Overall optimum found by optimizing segment 2 first
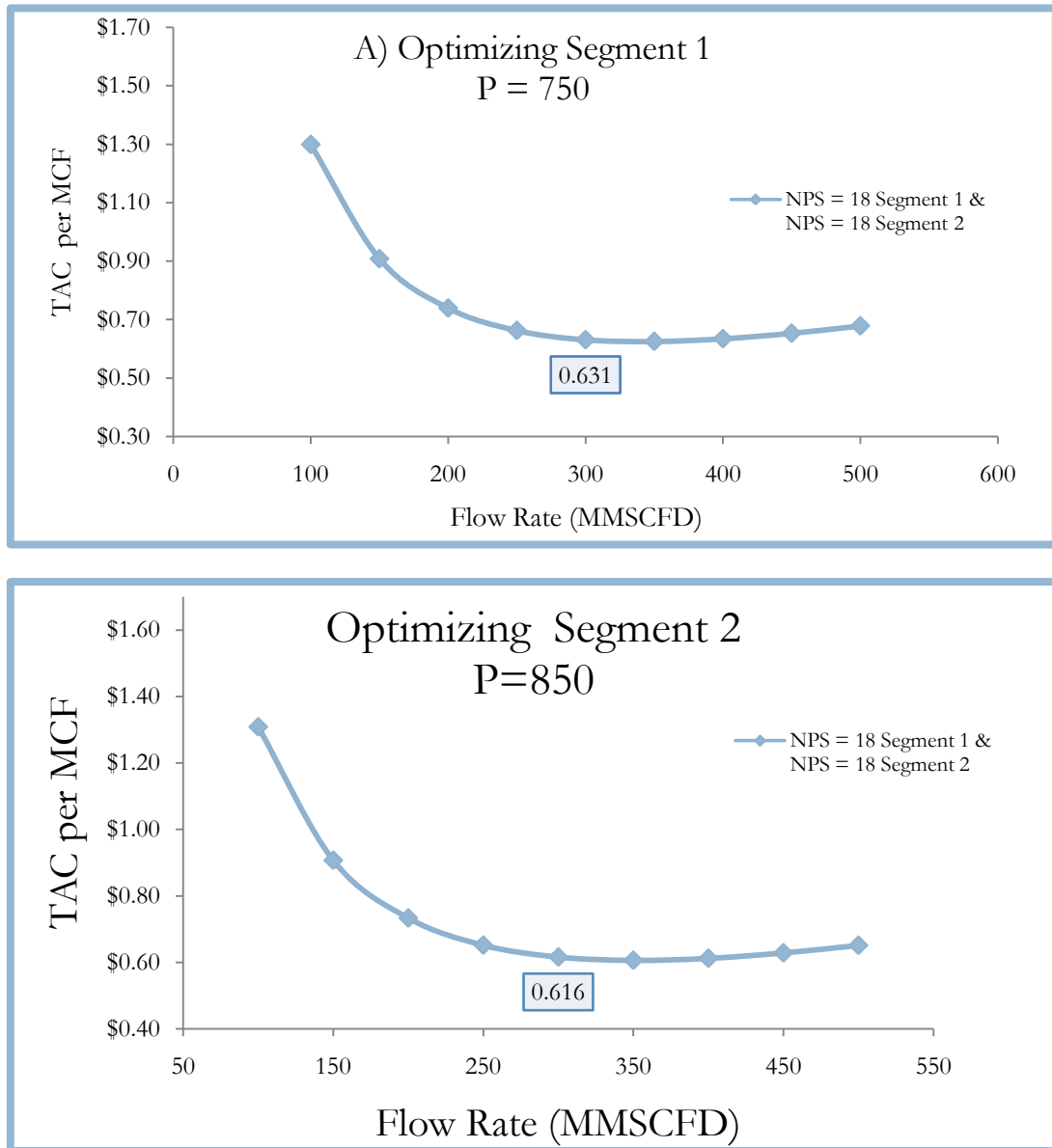
By optimizing segment 1 first and selecting a pressure of 750 psig and a diameter of 18 inches based on the optimum for segment 1, the optimum pipe diameter for segment 2 with that pressure is 18 inches. The Total Annual Cost per MCF is $ 0.631 for this configuration. But, by optimizing segment 2 first and selecting a pressure of 850 psig and a pipe diameter of 18 inches based on the optimum for segment 2, then the optimum diameter for segment 1 with that pressure is 18 inches. The TAC per MCF for this arrangement would be $ 0.607. It then becomes apparent that in order to get the optimum, all possibilities have to be examined in order to find the best optimum.
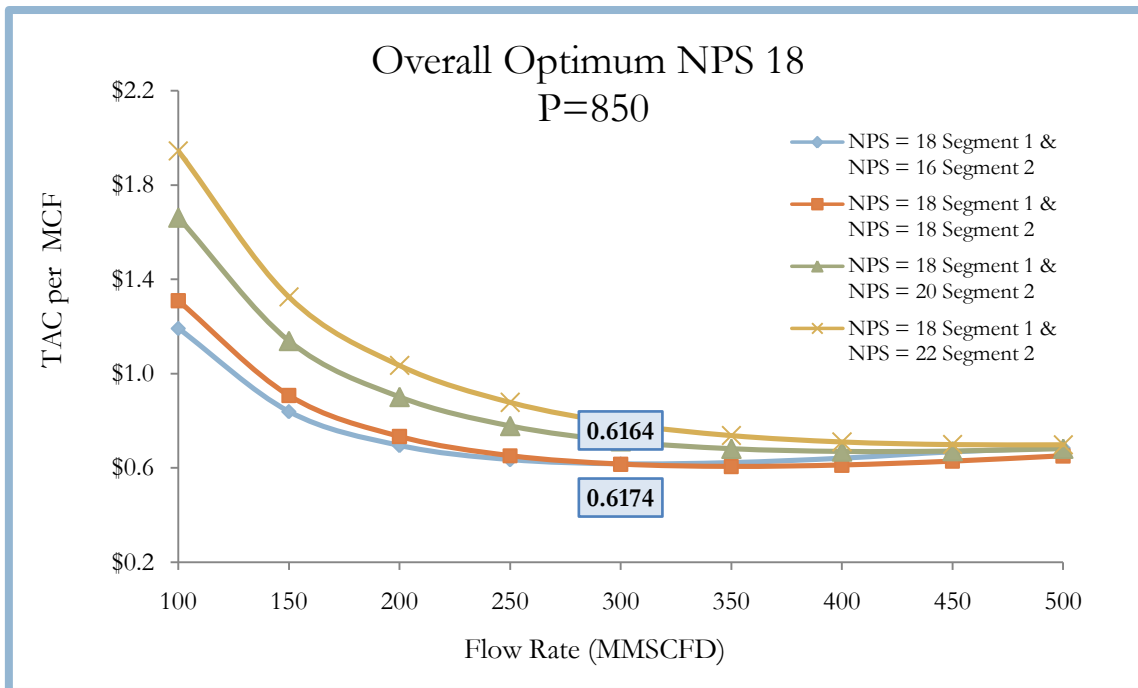
Figure 13: J-Curves of the total design.

Figure 13 above shows the overall optimum which occurs with a pipe size of 18 inches in both segments and an intermediate pressure of 850 psig. To find the overall optimum, there are 12 more curves for the intermediate pressure of 850 psig to account for all the diameter combinations. And then, there are 16 more curves for each of the two other intermediate pressures to come to a total of 48 J-curves to analyze.

This illustrates the importance of analyzing the network as a whole in order to determine the best combination of all parameters. For this simple two-pipe network, 18 total J-curves were required to analyze the system. For a larger network, it would be impossible to know which segment should be optimized first in order to arrive at the overall solution. As the number of segments grows, the number of simulations and J-curves required grows exponentially.

These unnecessary complications are time-consuming as well as misleading. For this simple two-pipe network, the following parameters influence the complexity:

1. 9 flow rates

2. 3 pressures

3. 4 diameters

This requires 432 simulations in order to arrive at a J-curve which does not provide accurate costs. A four-pipe segment would require 62,208 simulations which could easily take weeks to run the simulations and analyze the output for this very simple situation with no variable demand. Since most pipeline networks are far more complicated than a four-pipe segment, using Pro-II simulations to formulate J-curves is impractical and possibly inaccurate. The time required to perform this analysis increases exponentially for an increasingly complex network. This method does not even take into account variable demand during the project lifetime. The optimums are chosen for a specific flow rate, yet rarely does demand remain constant. A more inclusive method is required to evaluate even these simple pipeline networks accurately.

MATHEMATICAL PROGRAMMING MODEL

For the mathematical model, a correlation has to be used in the model to determine the pressure drop in a given pipe. This correlation is shown below.

$$\frac{Q^2 L}{D^5} = (P_1^2 - P_2^2)A - B\Delta Z$$

Where Q is the flow rate in M m^3 per day

     L is the length (km)

     D is the pipe inner diameter (in)

     P₂ is the outlet pressure (kPa)

     P₁ is the inlet pressure (kPa)

     ΔZ is the elevation difference (m)

A and B are determined by a regression analysis by graphing $Q^2 L/D^5$ (referred to as parameter F) versus $(P_1^2-P_2^2)$, and A will be the slope and B is the y intercept divided by $-\Delta Z$. Simulation software (Pro-II) was used to find $(P_1^2-P_2^2)$ for a range of Q, L and D.
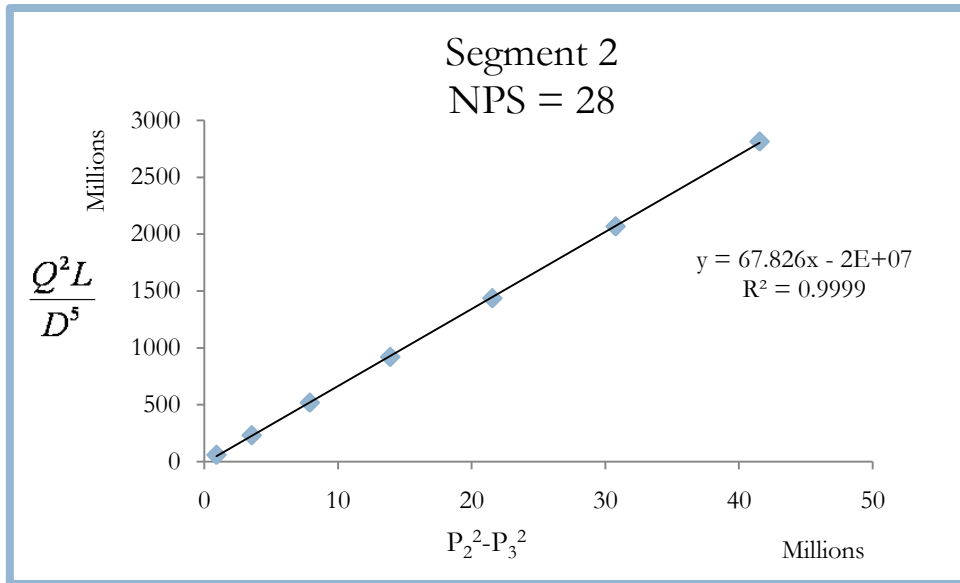
Figure 14: Deriving A and B values from Pro-II simulation data

The values for A and B were determined for different pipe diameters and for each pipe diameter for no elevation change, and a positive and a negative elevation change. So, for four pipe diameters, nine sets of A and B were found, which are shown in Figure 15 below.
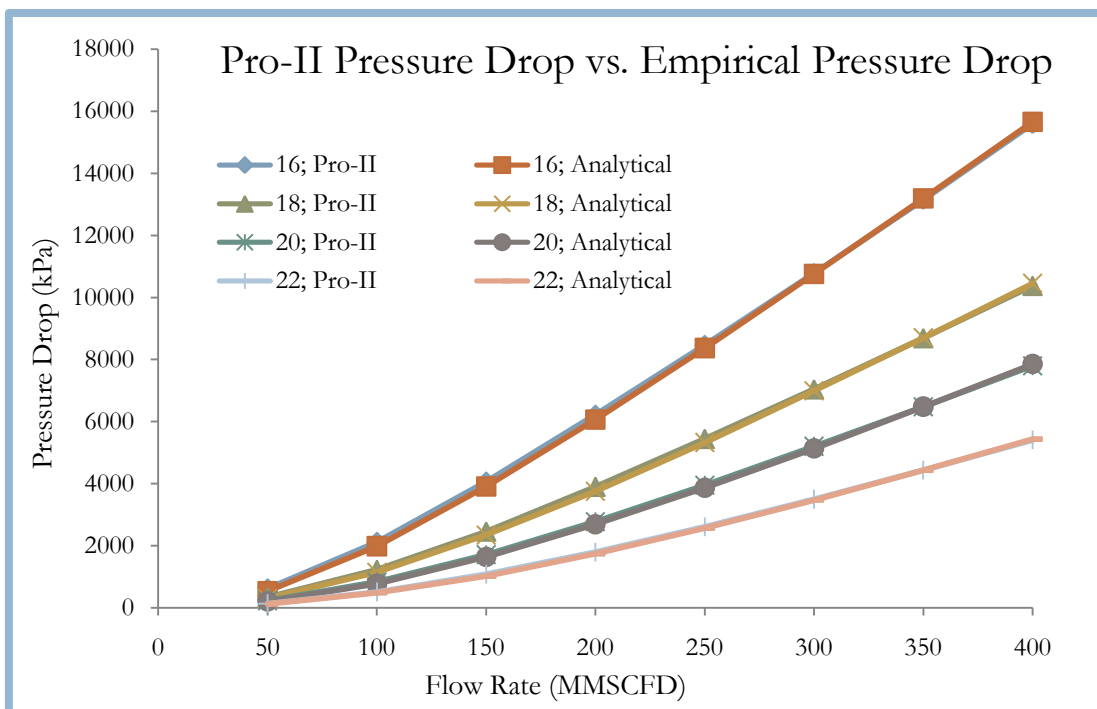
Figure 15: Pressure drops calcuated using A and B in the Empirical equation compared to pressure drops predicted by Pro-II simulation data.

There is a relatively small amount of error in the A and B factors for this flow rate, all less than 5 % error. This leads to the idea that there may be ranges of $(P_2^2-P_1^2)$ in which certain values of A and B are valid.

The advantage to using this correlation for finding the pressure drop, is that once the values for A and B are determined with sufficient accuracy, they may be used by the mathematical modeling software along with the same equations above that go from pressure drop to compressor work to compressor cost and pipeline cost to finally select the optimum pipe diameter and compressor for each segment. Then, whichever conditions are selected by the mathematical model can be used in simulation software to find a more accurate pressure drop and a more accurate work required by each compressor.

5. *EXAMPLES*

Before we use the model to solve larger problem, the two cases (single pipe and two segments pipeline) analyzed by J-curves will be also solved by the proposed mathematical model. Then two larger examples are presented: a linear network and a branched network.

5.1 - Single Pipe Case

The mathematical model was run for a single pipe segment of 120 mi with a flow rate of 200, 300 and 400 MMSCFD. The model predicted the following:

Mathematical Model Pressure Drop Error			
Q MMSCFD	dP Model (kPa)	dP Pro-II (kPa)	% Error
200	4975	5055	1.62
300	7425	7326	1.34
400	9950	9845	1.05

Table 3

In order to determine the accuracy of the mathematical model, the conditions of the optimal results predicted by the model were input into Pro-II simulation software. All conditions and parameters were maintained to ensure comparability. The resulting pressure drops predicted by simulations are compared in the table above, resulting in minimal errors. This ensures the accuracy and reliability of results obtained through the mathematical model.

5.2 – Two Segments Pipeline

The mathematical model was then used to optimize the two-pipe network discussed previously. The model considered four pipe diameters and optimized the network under the following conditions:

Non Linear Model – 2 Pipe Network		
	Pipe 1	Pipe 2
Pipe Diameter (in)	22	22
Compressor Work (hp)	10,740	0
Pressure Drop (psi)	1,830	1,490
TAC Model	$ 0.596	
TAC J-Curves	$ 0.616	

Table 4

The model was able to predict the optimum pipe diameters for each segment, along with the size of the required compressors and their locations. All consumer points were considered for adding a compressor station, yet the model predicted the need for only one compressor at the supply point.
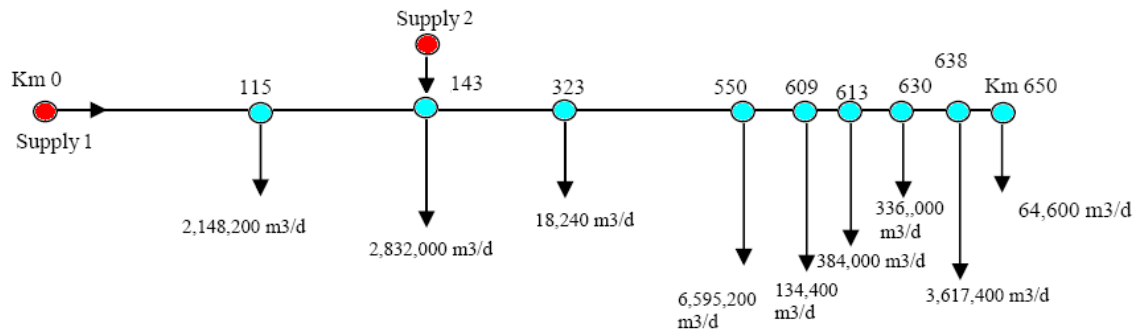
5.3 – Linear Pipeline

Figure 16

The following problem is solved applying the different proposed models and the solutions are presented and discussed: *Consider one wants to find the most profitable pipeline planning to delivery natural gas to nine consumers from two different supplier points over a twenty years life time project.*

The gas is composed by 91.9% of methane, 5% of ethane, 2% of propane, 1% of n-butane and 0.1% of n-pentane.

Table 5 presents the distances between supplier and consumers, and Table 6 presents the distances between consumers.

Table 5 – Distances between suppliers and consumers.

Distance (km)	Cons 1	Cons 2	Cons 3	Cons 4	Cons 5	Cons 6	Cons 7	Cons 8	Cons 9
Supplier 1	115	143	323	550	609	613	630	638	650
Supplier 2	28	0	180	407	466	470	487	495	507

Table 6 – Distances between consumers.

Distance (km)	Cons 1	Cons 2	Cons 3	Cons 4	Cons 5	Cons 6	Cons 7	Cons 8	Cons 9
Cons 1	0	28	208	435	494	498	515	523	535
Cons 2	28	0	180	407	466	470	487	495	507
Cons 3	208	180	0	227	286	290	307	315	327
Cons 4	435	407	227	0	59	63	80	88	100
Cons 5	494	466	286	59	0	4	21	29	41
Cons 6	498	470	290	63	4	0	17	25	37
Cons 7	515	487	307	80	21	17	0	8	20
Cons 8	523	495	315	88	29	25	8	0	12
Cons 9	535	507	327	100	41	37	20	12	0

Table 7 – Elevation at the suppliers and consumers.

Station	Elevation (m)
Supplier 1	42
Supplier 2	14.93
Consumer 1	7
Consumer 2	14.93
Consumer 3	60
Consumer 4	10
Consumer 5	120
Consumer 6	122
Consumer 7	235
Consumer 8	470
Consumer 9	890

Table 8 shows the conditions of the natural gas at the supplier stations, the current price, annual price increase and penalty agreed by contract.

Table 8 – Suppliers data.

Station	Pressure (kPa)	Temperature (ºC)	Price (US$/m^3)	Penalty (US$/m^3)
Supplier 1	1367	30	270	80
Supplier 2	1520	35	300	50

Table 9 – Consumers data.

Station	Demand (m^3/day)	Annual demand increasing	Price (US$/m^3)
Consumer 1	2,148,200	2%	440
Consumer 2	2,832,000	5%	400
Consumer 3	18,240	4%	390
Consumer 4	6,595,200	1%	420
Consumer 5	134,400	6%	445
Consumer 6	384,000	3%	460
Consumer 7	336,000	2%	420
Consumer 8	3,617,400	4%	410
Consumer 9	64,600	6%	450

Non Linear Model – 9 Pipe Network									
Pipe	1	2	3	4	5	7	6	8	9
Pipe Diameter (in)	36	36	36	32	32	24	24	24	24
Compressor Work (hp)	21,895	685	0	0	0	0	0	0	0

Table 10

The model created a network with expected results. The pipe diameters are highest at the beginning where the flow is greatest, and there are only two compressors at the first two points that should be built at the beginning of the project. The Total Annual Cost for this network is 142 million dollars per year. This model takes less than two minutes to run.
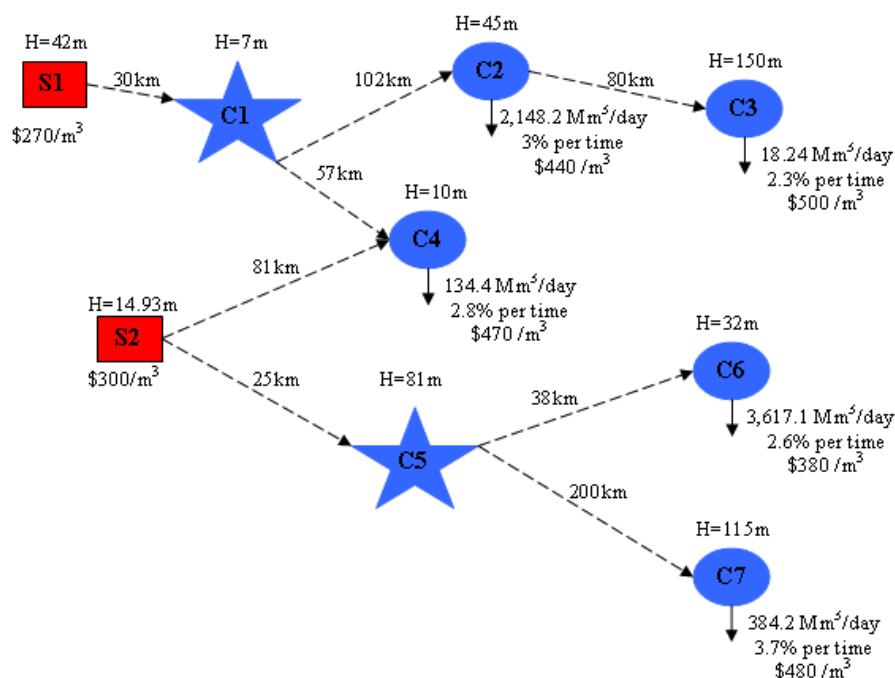
5.4 - Branched Pipeline Network

Figure 17

The conventional method of using J-curves to choose an optimum pipe diameter is time-consuming and introduces error into the analysis. When using Pro-II, the number of simulations is impractical for even four pipe segments. In most cases, networks are arranged in a complex form, such as the network shown below. This network consists of 8 pipes, three pipe diameters, and three pressures. To analyze a complicated network like this, it would take $4^8 3^7 9$ or 1.1 billion simulations, requiring approximately ten years working non-stop.

Figure 18

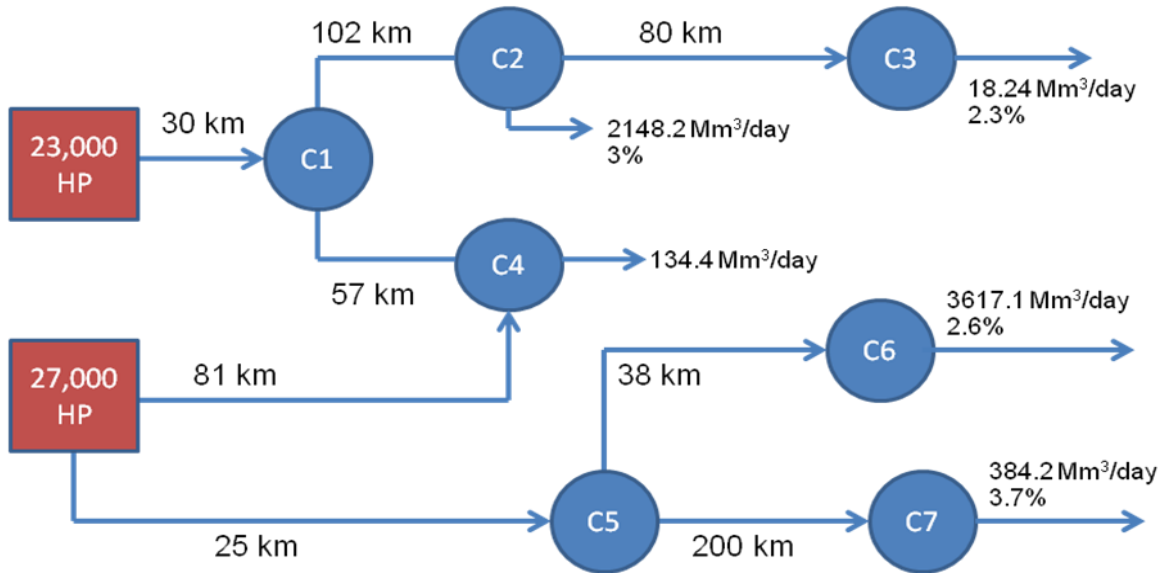The ramified network could account for increasing demand over time. The mathematical model could more easily handle a network larger than this, with even greater complexity.

Non Linear Model – Ramified Network							
Pipe	Pipe S1-C1	Pipe C1-C2	Pipe C2-C3	Pipe S2-S4	Pipe S2-S5	Pipe C5-C6	Pipe C5-C7
Pipe Diameter (in)	24	24	24	24	28	28	24

Compressor Work (hp)	5,010	0	0	8,350	8,350	0	0
Pressure Drop (psi)	65	4,190	4,255	180	30	100	10

Table 11

The model chose to put compressors at both supplier points at the beginning of the project and to only use pipe sizes 24 and 28. Another important note is that the model did not connect consumer points 1 and 4 with a pipe segment, indicating that excluding a connection results in a more economically optimized network.

5.5 Linear Mathematical Model-Results

A linear mathematical model was also considered for pipeline network optimization. This model relaxes the pressure parameters found in the non-linear model, allowing the upper and lower bounds of the system to be defined. In order to relax the pressures, the linear model discretizes the pressures into as many discrete units as precision demands.

The linear model was used to optimize the single pipe network discussed previously, with the results shown in Table 12.

Linear Mathematical Model – Single Pipe Segment					
	TAC (millions)	NPV (millions)	Time (sec)	Pipe S1-C1	NPS
Upper bound	63.5	-192.5	4.547	10932	22
Lower bound	47.6	-179.7	3.389	7922	22

Table 12

The upper and lower bounds predicted the range of possible solutions for the single pipe network. Both limits required a compressor at the supply.

The linear mathematical model was then used to optimize the two-pipe segment discussed previously. Table 13 shown below highlights the results.

	TAC (millions)	NPV (millions)	Time (sec)	Compressor Work Pipe 1	Compressor Work Pipe 2	NPS Pipe 1	NPS Pipe 2
Linear Mathematical Model – Two Pipe Segment							
Upper bound	87.98	-177.1	1219	15828	0	20	20
Lower bound	71	-184.3	1613	9242	3240	22	20

Table 13

From Table 13, it is apparent that the linear mathematical model predicts the requirement of two compressor stations, one at the supply and on at consumer point 1. The upper bound predicts only a single compressor station at the supply point. As expected, the upper bound predicted a higher Total Annual Cost per MCF.

The linear nine-pipe segment previously analyzed by the non-linear mathematical model was then analyzed by the linear model, with the following results:

	NPS Pipe 1	NPS Pipe 2	NPS Pipe 3	NPS Pipe 4	NPS Pipe 5	NPS Pipe 6	NPS Pipe 7	NPS Pipe 8	NPS Pipe 9
Linear Mathematical Model – Nine Pipe Segment									
Upper bound pipe size	32	28	28	28	24	28	24	24	24
Lower bound pipe size	32	28	28	28	24	28	24	24	24
Upper bound Compressor Work	19421	9556	0	8755	0	0	0	0	0
Lower bound Compressor Work	19421	9556	0	8755	0	0	0	0	0

Table 14

As shown in Table 14, the upper and lower bounds predicted the same optimized pipe diameters. The range of Total Annual Costs per MCF was found to be $376 - $476 million. Both limits predict the same required compressor station sizes and locations.

Finally, the ramified network previously discussed was optimized by the linear mathematical model, with the results shown in Table 15 below.

Linear Mathematical Model – Ramified Network								
Mathematical Model	Pipe S1-C1	Pipe C1-C2	Pipe C2-C3	Pipe C1-C4	Pipe S2-C4	Pipe S2-C5	Pipe C5-C6	Pipe C5-C7
Upper bound pipe size	24	24	24	None	24	36	24	24
Lower bound pipe size	24	24	24	None	24	36	24	24
Upper bound Compressor Work	7072.3	1690	13.85	0	12432	12432	0	0
Lower bound Compressor Work	4705.3	1690	13.85	0	8017	8017	0	0

Table 15

From the linear mathematical model, the range of Total Annual Costs per MCF was predicted to be $96 to $130 million. Both the upper and lower limits analysis predicted the same compressor station locations, but with varied compressor station sizes. The linear mathematical model predicted the exclusion of a pipe segment from consumer point 1 to consumer point 4, which is consistent with the prediction of the non-linear model.

The accuracy of the linear mathematical model's upper and lower bound predictions has not been verified. However, the non-linear mathematical model predicted network optimum conditions to a high degree of accuracy. Although, it should be noted that the mathematical models need to be used with great care because it is only as accurate as the input that the model is given.

6. EXTENSIONS

- Contracts:
  - Suppliers – It is being assumed that price and purchased amount of gas is previously agreed. Also, a penalty is applied when the agreed

amount of gas is not bought. This agreed amount is an important parameter and will be handled as a decision variable.

- o Consumers – The contract with consumers also define a price, and penalty due to not delivery the required demand. Demand will be handled as input parameter. In this contract, the penalty per cubic meter not delivered is 10% of the agreed price.

In both case, price readjustments (from prices forecast) were not directly considered since both sides will have the same inflation influence. However, the value of the money still be considered when a discount factor is applied.

7. CONCLUSIONS

In conclusion, J-Curves can be very time consuming and in some instances misleading. To accurately examine all the parameters together requires a very large amount of time and simulations that will only be applicable for that one situation. It is worthwhile to investigate a model that could be adaptable to many other designs. The mathematical model analyzed in this paper shows promise and could become a very useful tool for pipeline network design.

Nomenclature

Parameters

DF – Discount factor

VCC – Coefficient related with the variable part of compressors capital cost

FCC – Coefficient related with the fixed part of compressors capital cost

PC – Parameter related with the cost of pipe of different diameters

SPrice – Prices charged by each supplier

CPrice – Prices charged for each consumer

OPC – Compressors operating cost

OPH – Annual operating hours

LCC – Distance between two consumers

LSC – Distance between supplier and consumer

ID – Inside diameters options

A – Pre pressure drop parameter in the empirical equation

B – Pre elevation parameter in the empirical equation

ZC – Elevation of each consumer

ZS – Elevation of each supplier

UB – Flowrate upper bound at each time (defined by the summation of demands each year)

$Demand$ – Demand required by each consumer in each year

$CPenal$ – Penalty applied when demand is not delivered

$SPenal$ – Penalty applied when the agreed amount is not bought

k – Cp/Cv

ST – Suction temperature at each supplier

SP – Suction pressure at each supplier

Z – Compressibility factor, dimensionless

η – Compressor efficiency

Q – volumetric flow rate (SCFD)

E – pipeline efficiency

P_b – base pressure (psia)

T_b – base temperature (°R)

P_1 – upstream pressure (psia)

P_2 – downstream pressure (psia)

T_f the average gas flow temperature (°R)

L_e the equivalent length of pipe segment (miles)

G – gas gravity
e = base of natural logarithms (e=2.718…)

s = elevation adjustment parameter, dimensionless

H_1 is the upstream elevation (m)

H_2 is the downstream elevation (m)

T^{amb} – Ambient temperature

ρ^{sta} – Density at standard conditions

C – Empirical constant of erosion velocity equation

α – Angular coefficient of the empirical equation for density

β – Linear coefficient of the empirical equation for density

Variables

NPV – Net present value

Revenue - Annual revenue

FCI – Annual fixed capital of investment

Oper – Operating cost at each year

Penal – Penalties paid at each year

WS – Work done by the compressor at each supplier at each year

WC – Work done by the compressor at each consumer station at each year

DP – Discharge pressure of each compressor at the supplier station

Caps – Compressor capacity at each supplier point

Capc – Compressor capacity at each consumer point

Agreed – Amount of gas agreed to purchase from each supplier

XDCC – Binary variable related to the diameter size and installation time of a pipeline between two consumers points

XDSC – Binary variable related to the diameter size installation time of a pipeline between a supplier point and a consumer point

XCC – Binary variable related to time when a compressor is installed at the consumer

XCS – Binary variable related to time when a compressor is installed at the supplier

YDCC – Binary variable related to the operation conditions of the pipeline between two consumers points at each time

YDSC – Binary variable related to the operation conditions of the pipeline between a supplier and a consumer at each time

YCC – Binary variable related to the operation conditions of the compressor installed at the consumer

YCC – Binary variable related to the operation conditions of the compressor installed at the supplier

XPC – Binary variable related the pressure at each consumer at each time

QC – Flowrate delivered at each consumer point

QSC – Flowrate between a supplier point and a consumer point

QCC – Flowrate between a two consumer points

Parameters

PXCC – Binary parameter related to the allowed existence of a compressor at consumer points

PXSC – Binary parameter related to the allowed existence of a compressor at supplier points

XPCC – Binary parameter related to the allowed existence of a pipeline between two consumers points

XPSC – Binary parameter related to the allowed existence of a pipeline between a supplier point and a consumer point

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